Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE EXAMINATION, 2016

APPLIED MATHEMATICS – ORDINARY LEVEL

FRIDAY, 24 JUNE – AFTERNOON, 2:00 to 4:30

Six questions to be answered. All questions carry equal marks.

A *Formulae and Tables* booklet may be obtained from the Superintendent.

Take the value of g to be 10 m s^{-2}.

$\vec{i}$ and $\vec{j}$ are unit perpendicular vectors in the horizontal and vertical directions, respectively, or eastwards and northwards, respectively, as appropriate to the question.

Marks may be lost if necessary work is not clearly shown.

Marks may be lost for omission of correct units with numerical answers.

1. The points P and Q lie on a straight level road.

A car travels along the road in the direction from P to Q. It is initially moving with a uniform speed of 14 m s^{-1}. As it passes P it accelerates uniformly for 8 seconds until it reaches a speed of 30 m s^{-1}.

Then the car decelerates uniformly from a speed of 30 m s^{-1} to a speed of 22 m s^{-1}. The car travels 52 metres while decelerating.

It now continues at a constant speed of 22 m s^{-1} for 10 seconds and then passes Q.

(a) Draw a speed-time graph of the motion of the car from P to Q.

(b) Find **(i)** the acceleration

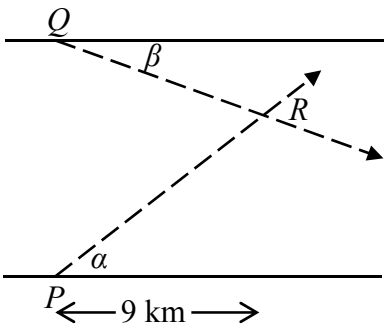
(ii) the deceleration

(iii) $|PQ|$, the distance from P to Q

(iv) the average speed of the car as it travels from P to Q

(v) the time for which the car is moving at or above its average speed.

2. P is a point on the southern bank of a river.
Q is a point directly opposite P on the northern bank.



Ship A departs from P at a constant speed of 52 km h^{-1} and travels in a direction East $\alpha°$ North, where $\tan\alpha = \frac{12}{5}$.

Ship B departs from Q at a constant speed of 51 km h^{-1} and travels in a direction East $\beta°$ South, where $\tan\beta = \frac{8}{15}$.

Find **(i)** the velocity of A in terms of $\vec{i}$ and $\vec{j}$

(ii) the velocity of B in terms of $\vec{i}$ and $\vec{j}$

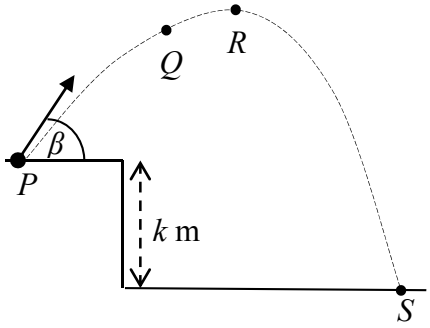
(iii) the velocity of A relative to B in terms of $\vec{i}$ and $\vec{j}$.

The paths of A and B intersect at point R, which is 9 km downstream from P and Q.

Find **(iv)** the time it takes B to reach R and how much longer it takes A to reach R.

(v) the width of the river, assuming its banks are parallel.

3. A particle is projected from a point P, as shown in the diagram, with an initial speed of 74 m s^{-1} at an angle β to the horizontal, where $\tan\beta = \frac{35}{12}$.



The particle reaches point Q after 4 seconds of motion.
R is the highest point reached by the particle.

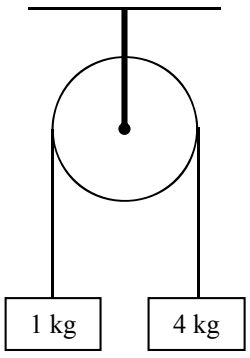
Find **(i)** the initial velocity of the particle in terms of $\vec{i}$ and $\vec{j}$

(ii) the velocity of the particle at point Q in terms of $\vec{i}$ and $\vec{j}$

(iii) the displacement of R from P in terms of $\vec{i}$ and $\vec{j}$

(iv) the value of k, given that the particle reaches S after 16 seconds of motion.

4. **(a)** Masses of 1 kg and 4 kg are connected by a taut, light, inextensible string which passes over a smooth light fixed pulley.



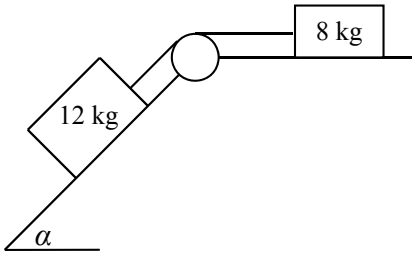
The system is released from rest.

Find **(i)** the common acceleration of the masses

(ii) the tension in the string.

(b) Masses of 8 kg and 12 kg are connected by a taut, light, inextensible string which passes over a smooth light fixed pulley as shown in the diagram.

The 8 kg mass lies on a rough horizontal plane and the coefficient of friction between the 8 kg mass and the plane is $\frac{3}{4}$.
The 12 kg mass lies on a smooth plane which is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{4}{3}$.
The system is released from rest.



(i) Show on separate diagrams the forces acting on each mass.

(ii) Find the common acceleration of the masses.

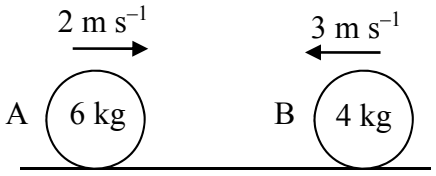
(iii) Find the tension in the string.

(iv) Find the common speed of the masses after two seconds of motion.

5. **(a)** A smooth sphere A, of mass 6 kg, collides directly with another smooth sphere B, of mass 4 kg, on a smooth horizontal table.

Spheres A and B are moving in opposite directions with speeds of 2 m s^{-1} and 3 m s^{-1} respectively.

The coefficient of restitution for the collision is $\frac{2}{5}$.



Find **(i)** the speed of A and the speed of B after the collision
(ii) the loss in kinetic energy due to the collision
(iii) the magnitude of the impulse imparted to A due to the collision.

(b) A ball is fired vertically down with a speed of 2 m s^{-1} from a height of 3 metres onto a smooth horizontal floor. The ball hits the floor and rebounds to a height of 1·8 metres.

The coefficient of restitution between the ball and the floor is e.

Find **(i)** the speed of the ball when it hits the floor
(ii) the value of e.

6. **(a)** Particles of weight 9 N, 8 N, q N and 2 N are placed at the points $(-4, 3)$, $(8, 6)$, $(p, 5)$ and $(q, -p)$ respectively.

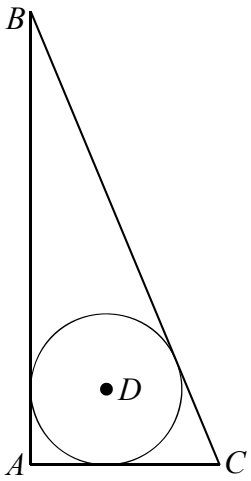
The co-ordinates of the centre of gravity of the system are $(p, 4)$.

Find **(i)** the value of p
(ii) the value of q.

(b) A triangular lamina with vertices A, B and C has the portion inside its incircle removed. D is the centre of the incircle.

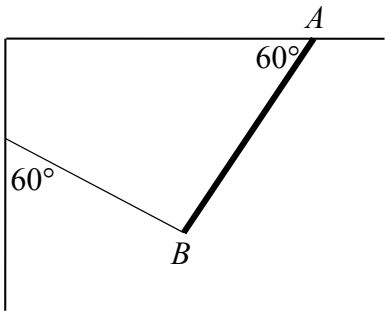
The co-ordinates of the points are $A(0, 0)$, $B(0, 108)$, $C(45, 0)$ and $D(18, 18)$.

Find the co-ordinates of the centre of gravity of the remaining lamina.



7. A uniform rod, AB, of length 4 m and weight 160 N is smoothly hinged at end A to a horizontal ceiling.

One end of a light inextensible string is attached to B and the other end of the string is attached to a vertical wall.



The rod makes an angle of 60° with the ceiling and the string makes an angle of 60° with the wall, as shown in the diagram.

The rod is in equilibrium.

(i) Show on a diagram all the forces acting on the rod AB.

(ii) Write down the two equations that arise from resolving the forces horizontally and vertically.

(iii) Write down the equation that arises from taking moments about the point A.

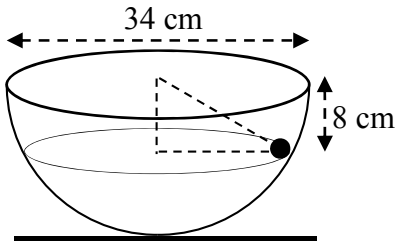
(iv) Find the tension in the string.

(v) Find the magnitude of the reaction at the point A.

8. **(a)** A particle describes a horizontal circle of radius 1·5 metres with uniform angular velocity ω radians per second. Its speed is 3 m s^{-1} and its mass is 2 kg.

Find **(i)** the value of ω
(ii) the time to complete one revolution
(iii) the centripetal force on the particle.

(b) A hemispherical bowl of diameter 34 cm is fixed to a horizontal surface.



A smooth particle of mass 1 kg describes a horizontal circle of radius r cm on the smooth inside surface of the bowl.

The plane of the circular motion is 8 cm below the top of the bowl.

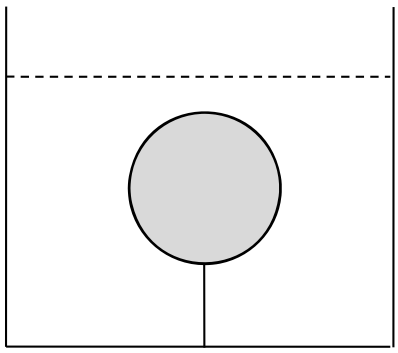
Find **(i)** the value of r
(ii) the reaction force between the particle and the surface of the bowl
(iii) the angular velocity of the particle.

9. **(a)** A solid piece of metal has a weight of 23 N.
When it is completely immersed in water, the metal appears to weigh 17 N.

(i) State the principle of Archimedes.
(ii) Find the volume of the metal.
(iii) Find the density of the metal.

[Density of water = 1000 kg m^{-3}]

(b) A solid sphere has a radius of 5 cm.



The density of the sphere is 800 kg m^{-3} and it is completely immersed in a tank of liquid of density 1200 kg m^{-3}.

The sphere is held at rest by a light inextensible, vertical string which is attached to the base of the tank.

Find the tension in the string.

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